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Drag factor – how it differs from dynamic coefficient of friction

The drag factor of a wheel, f, is related to but differs from μ, the dynamic coefficient of friction. f is linearly related to μ.

$$f = \mu \cdot n \pm m$$

where n is the braking efficiency of the wheel and m is the slope across which the wheel slides. E.g. if the surface has a 4% slope, one would use $m = 0.04$. If the vehicle slides up, use +0.04. If it slides down the slope, use -0.04. The braking efficiency is an adjustment depending upon whether the wheel is locked or not. If the wheel is free to roll and the vehicle is spinning, n adjusts for this. Both of these adjustments are explained below.

Adjustment for skid on a non-level surface

Consider the case shown in Figure 1. The automobile is skidding up a slope, m. The figure shows the free-body diagram and the mass-acceleration diagram for the automobile.

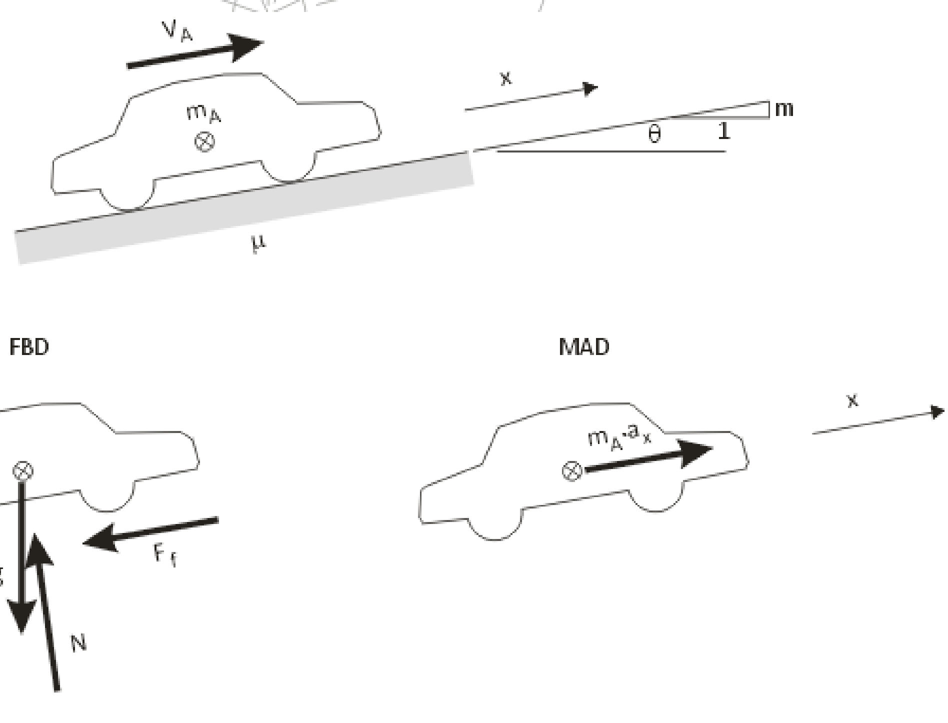


Figure 1 – Vehicle skidding up a slope

The x-equation of motion is

$$\sum F_x: \qquad -F_f - m_A \cdot g \cdot \sin(\theta) = m_A \cdot a_x$$

Also, the friction force is

$$F_f = m_A \cdot g \cdot \mu \cdot n$$

where *n is used to adjust* μ for braking efficiency (see below). Thus

$$-m_A \cdot g \cdot \mu \cdot n - m_A \cdot g \cdot \sin(\theta) = m_A \cdot a_x$$

and

$$-m_A \cdot g \cdot [\mu \cdot n + \sin(\theta)] = m_A \cdot a_x$$

Here the retarding force is shown on the left-hand side of the equation. The friction force is some ratio of the vertical force. We see that the vertical force is adjusted by $\mu \cdot n + \sin(\theta)$, so this is the drag factor for a vehicle sliding up a slope.

$$f = \mu \cdot n + m$$

So

$$m = \sin(\theta) \approx \tan(\theta)$$

for small θ.

The car sliding down a slope is shown in Figure 2.

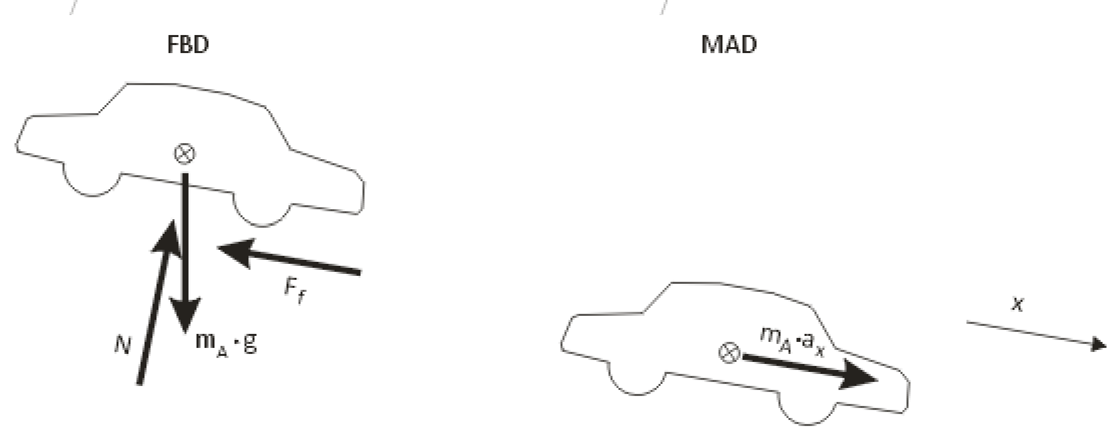


Figure 2 – Vehicle skidding down a slope

The x-equation of motion is then

$$\sum F_x: \qquad -F_f + m_A \cdot g \cdot \sin(\theta) = m_A \cdot a_x$$

So

$$-m_A \cdot g \cdot \mu \cdot n + m_A \cdot g \cdot \sin(\theta) = m_A \cdot a_x$$

and

$$-m_A \cdot g \cdot [\mu \cdot n - \sin(\theta)] = m_A \cdot a_x$$

Thus

$$f = \mu \cdot n - m$$

Adjustment for braking efficiency

The coefficient of friction, μ, is multiplied by the braking efficiency, *n*, to adjust it for the case where a wheel is not locked. A common adjustment is for the case where the wheel is completely free to rotate but the car is spinning about its vertical axis. Figure 3 shows a wheel with its important directions.

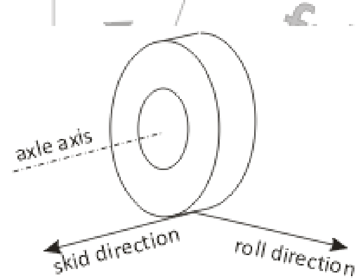


Figure 3 – Important, wheel-related directions

If the unlocked wheel is travelling in the roll direction, there is very little resistance to motion. But if the wheel is moving in the skid direction, it must slide across whatever surface it is on without rolling. In that case, there is no rolling motion, so the force of friction (F_f) is simply $\mu \cdot F_w$, where F_w is the normal force of the tire on the ground. If the direction of travel is somewhere in between these two axes, the wheel motion will be partially rolling and partially skidding. So the effective coefficient of friction, the drag factor *f*, will be between 0 and μ. In a skid, however, the vehicle may be spinning about a vertical axis through its center of mass. So the direction of the wheel velocity relative to these axes will be constantly changing. This situation is handled conventionally by calculating an average drag factor based upon the total angular displacement of the vehicle during the skid.

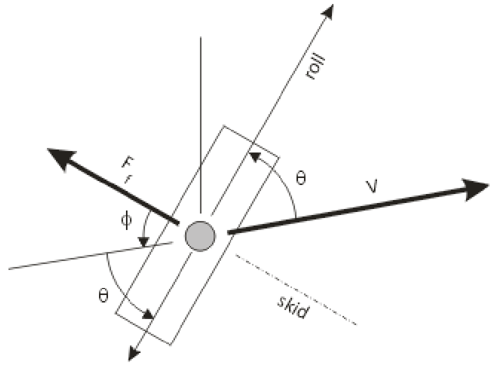


Figure 4 – Plan view of a wheel of a skidding vehicle

Figure 4 shows a plan view of an unlocked wheel in a skid. The velocity of the contact patch of the tire with the surface is oblique, so not oriented in either the roll or the skid directions. The friction force, F_f, is the normal force of the wheel on the ground times μ, the dynamic coefficient of friction.

$$F_f = F_w \cdot \mu$$

The velocity of the vehicle immediately after impact will be calculated by applying the energy principle for the duration of the post-collision spin-out. Thus the work of the friction force determines the energy lost in the post-collision skid. In Figure 4 the only component of the friction force that does work is the component opposed to the motion. So

$$\Delta W = F_f \cdot \cos(\varphi) \cdot \Delta d = F_w \cdot \mu \cdot \cos(\varphi) \cdot \Delta d$$

where Δd is the incremental displacement in the direction of the velocity. From a trigonometric identity, $\cos(\varphi) = \sin(\theta)$, so

$$\Delta W = F_w \cdot \mu \cdot \sin(\theta) \cdot \Delta d$$

As the vehicle spins, θ varies from an initial angle, θ_i, to a final angle, θ_f. Thus sin(θ) varies from sin(θ_i) to sin(θ_f). The conventional simplification of this is to calculate a linear distribution of sin(θ) between θ_i and θ_f and then use the average of these sines, multiplied times μ, to get a drag factor for the spin-out. So

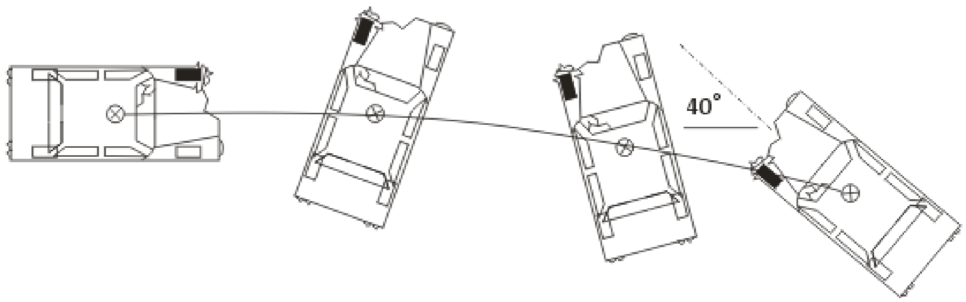
$$f = avg\left[\sin(\theta_i) \ldots \sin(\theta_f)\right] \cdot \mu$$

Note that θ is the angle between the velocity and the roll direction of the wheel.

Examples

Example 1

An automobile undergoes a rotating spin-out after a collision as shown in the drawing below. The spin-out starts from the left. The right-most position is the end, stationary position of the automobile after crash. The roadway surface at this point is level.



Due to the collision on the left-front corner, the left-front wheel is locked during the collision. The other three wheels can freely rotate. The asphalt pavement is dry for the collision, so μ = 0.8. The length of the curved trajectory shown is 9.75 meters. Assume the automobile has a 50-50, front/rear weight distribution. Calculate the velocity of the automobile at the beginning of the spin-out trajectory.

Solution

Since the automobile undergoes a 140°-rotation on level pavement, the rotating wheels will need an adjustment due to the angle of rotation between 0° and 140°. A table is created with θ and sinθ shown between 0° and 140°.

θ (deg)	sin θ
0	0.0000
10	0.1736
20	0.3420
30	0.5000
40	0.6428
50	0.7660
60	0.8660
70	0.9397
80	0.9848
90	1.0000
100	0.9848
110	0.9397
120	0.8660
130	0.7660
140	0.6428
Avg	0.6943

This average sine value is applied to μ, so

$$f = 0.6943 \cdot 0.8 = 0.555$$

This drag factor applied to the three, unlocked wheels. The locked wheel uses the entire μ for its drag factor, since the locked wheel resists motion equally, no matter what its orientation relative to the skid direction. Thus

$$\frac{1}{2} m_A \cdot V_{Af}^2 = m_A \cdot g \left(\frac{3}{4} f + \frac{\mu}{4} \right) d_{Af}$$

$$V_{Af} = \sqrt{2 \cdot g \left(\frac{3}{4} f + \frac{\mu}{4} \right) d_{Af}}$$

$$V_{Af} = \sqrt{2 \cdot 9.81 \frac{m}{sec^2} \left(\frac{3}{4} \cdot 0.555 + \frac{0.8}{4} \right) 9.75\, m} = 10.9 \frac{m}{sec} \cdot \frac{km}{1000\, m} \cdot \frac{3600\, sec}{hr} = 39.1 \frac{km}{hr}$$

Example 2

Assume that the same accident occurs but on a surface that has a 2% downward slope between the start position of the spin-out and the end position of the automobile. Calculate the post-collision velocity under these conditions.

Solution

The slope adjustment applies to both drag factors, i.e. to the locked-wheel drag factor (f_L) and to the unlocked-wheel drag factor (f_U).

$$f_L = \mu - m = 0.8 - 0.02 = 0.78$$

$$f_U = \mu \cdot n - m = 0.555 - 0.02 = 0.535$$

So

$$V_{Af} = \sqrt{2 \cdot g \left(\frac{3}{4} f_U + \frac{1}{4} f_L \right) d_{Af}} = \sqrt{2 \cdot 9.81 \frac{m}{sec^2} \left(\frac{3}{4} 0.535 + \frac{1}{4} 0.78 \right) 9.75\, m} \cdot \frac{km \cdot 3600\, sec}{1000\, m \cdot hr}$$

$$V_{Af} = 38.4 \frac{km}{hr}$$

This is a little slower than the calculation for a flat surface. That makes sense, since the starting speed would not have to be as great for the vehicle to skid the same distance on a downward slope.

Note that the drag factor adjustments are made to μ. μ is selected to fit the pavement conditions. So if the pavement is wet, for example, one uses a lower μ. One does not make an adjustment to μ in calculating the drag factor.