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Coriolis acceleration

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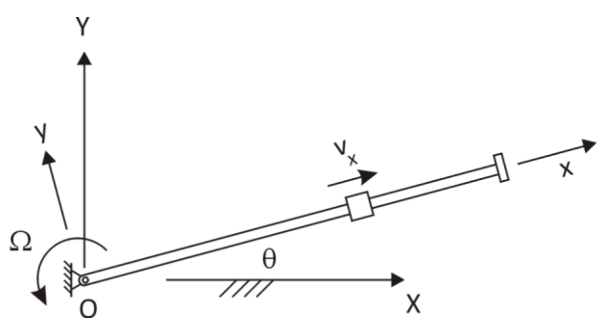
By derivation of the five-term acceleration equation with a spinning and translating reference frame, we see that there are two terms that appear when an object moves inside this rotating frame. One of these is Coriolis acceleration, which was not well known until long after Newton worked out his three laws and Euler applied them to rigid bodies. The Coriolis term is

$$\vec{a}_{Cor} = 2\vec{\Omega} \times \vec{v}_{xy}$$

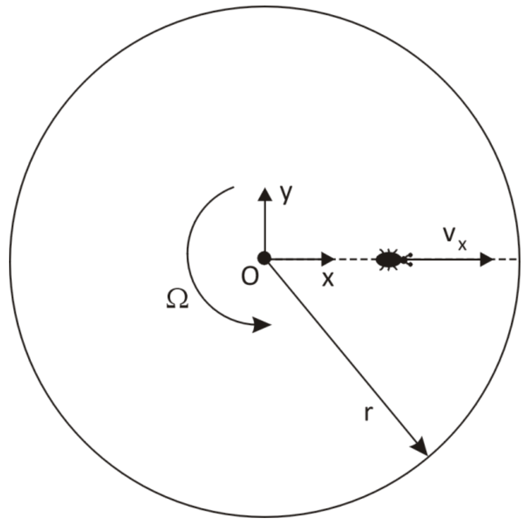
How this term arises mathematically when there is movement in a rotating frame is the topic of another article (see www.aoengr.com/Dynamics/RotatingReferenceFrame.pdf). This article explains Coriolis acceleration from a more pragmatic standpoint, to allow you to develop your intuition a bit regarding this non-intuitive concept.

First note that the components of Coriolis acceleration involve only two velocities: 1) the rotational velocity of the rotating frame, usually attached to a rotating body, and 2) the velocity of an object within this rotating frame. A simple case would be a rotating rod with a collar moving along the rod as shown in the figure below. Thus, like normal acceleration, Coriolis acceleration results from velocity.



Rotating rod with sliding collar

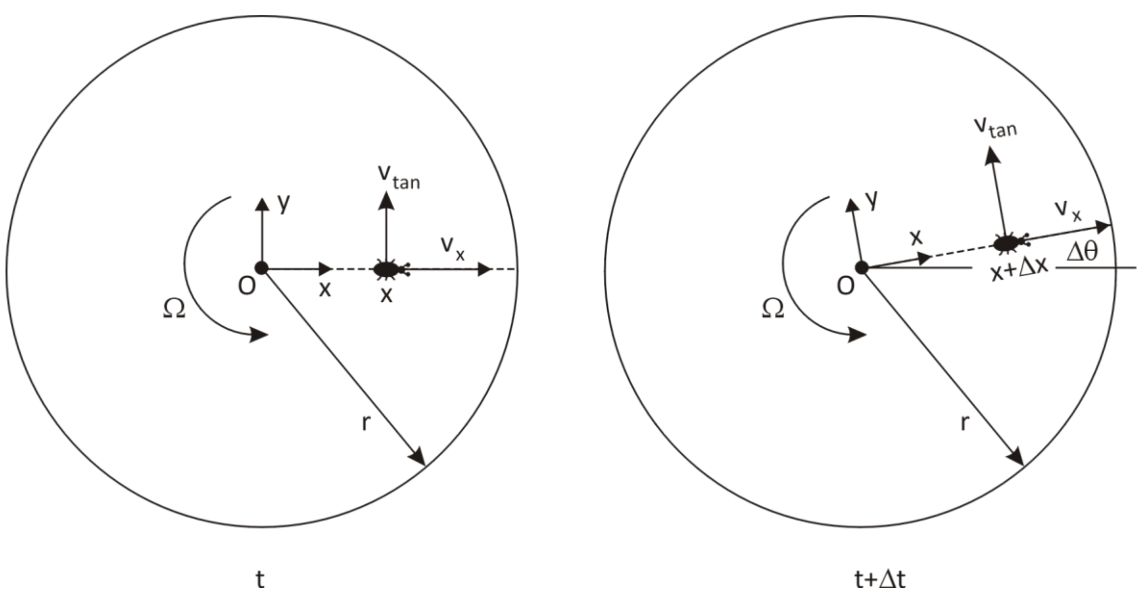
To explain Coriolis and give some examples of how it manifests itself, I'd like to avail myself of a scenario that was used to explain Coriolis to me, when I was studying engineering in Mississippi in the middle 1970s. This is a cockroach walking on a vinyl LP record as shown in the figure below.



Cockroach walking out on a rotating LP platter

Let's consider the simple situation where the rotational speed Ω is constant and the cockroach's walking velocity is also constant. Obviously as the cockroach walks outward, his distance from the center of rotation increases, and therefore his velocity due to the rotation increases. Let's look at this in detail.

The figure below shows the cockroach at an instant in time (t) and then at another instant shortly thereafter ($t+\Delta t$). The disk has turned through a small angle $\Delta\theta$, and the cockroach has moved out a tiny amount Δx. At t the tangential velocity v_Ω is $\Omega * x$. At $t+\Delta t$, because of the increased radius, the tangential velocity v_{tan} has increased to $\Omega *(x+\Delta x)$. This increase in v_{tan} is part of the Coriolis acceleration.



Cockroach's path and tangential speed after a short interval

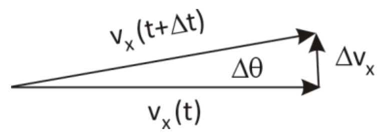
Thus

$$a_{Cor-\Delta x} = \frac{v_{tan}(t+\Delta t) - v_{tan}(t)}{\Delta t} = \frac{\Omega * (\mathrm{x} + \Delta\mathrm{x}) - \Omega * \mathrm{x}}{\Delta t} = \Omega \frac{\Delta x}{\Delta t} = \Omega * v_x$$

If we make Δt ever shorter and, thus, $\Delta\theta$ ever smaller, we can see that the direction of this acceleration is tangential, i.e. in the *y*-direction. Thus

$$\vec{a}_{Cor-\Delta x} = \Omega v_x \hat{j}$$

There is another change that must also be taken into account. A vector can change in time by increasing or decreasing in magnitude but also by changing direction. The cockroach's velocity on the disk has changed direction in this instant, though, for this case, the velocity is constant. The figure below shows a close-up of this change, with the two vectors placed tail-to-tail.



Change in v_x's direction

As we make Δt and $\Delta\theta$ ever smaller, Δv_x's length approaches that of the arc length of the rotated vector. Thus

$$\Delta v_x = \Delta\theta * v_x$$

The acceleration due to this effect is thus

$$a_{Cor-\Delta\theta} = \frac{\Delta v_x}{\Delta t} = \frac{\Delta\theta * v_x}{\Delta t} = \Omega * v_x$$

exactly the same result as before, due to the increase in tangential velocity. It is also easy to see that with a very small $\Delta\theta$, the direction of this acceleration is exactly the same as that of $\vec{a}_{Cor-\Delta x}$, that is in the *y*-direction. Thus

$$\vec{a}_{Cor-\Delta\theta} = \Omega v_x \hat{j}$$

The total Coriolis acceleration is thus the combination of these two effects.

$$\vec{a}_{Cor} = \vec{a}_{Cor-\Delta x} + \vec{a}_{Cor-\Delta\theta} = 2\Omega v_x \hat{j}$$

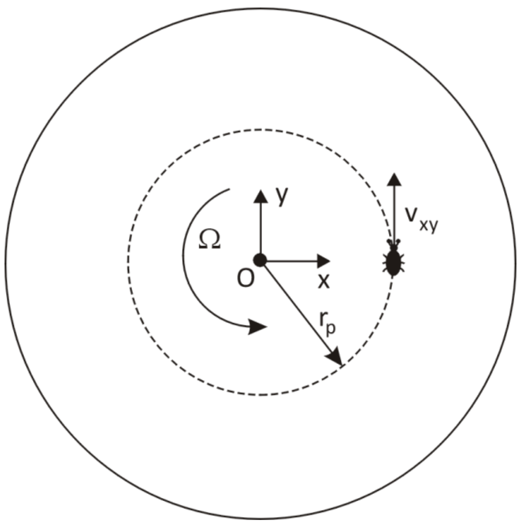
Note that this is precisely the same as the cross product that results from deriving the five-term acceleration equation.

$$\vec{a}_{Cor} = 2\vec{\Omega} \times \vec{v}_{xy}$$

This equation is more general because we are not constraining the cockroach just to walk on a straight path along the *x*-axis. The same type of analysis could be done for any random path with the same result.

One thing interesting about Coriolis acceleration that is shown by the cross product is this: If the disk is rotating counter-clockwise, so that $\vec{\Omega} = \Omega\hat{k}$, the Coriolis acceleration is always directed to the left of the path taken by the cockroach. With clockwise rotation, Coriolis acceleration is 90° to the right of the path.

Some interesting cases



Cockroach walking on circular path in direction of rotation

At left the cockroach has turned and is walking in a circular path around the platter. With this, his tangential speed will increase by v_{xy} from what it would be due to Ω alone. That is

$$v_{tan} = \Omega r_p + v_{xy}$$

If a light were put on the bug's back and the lights were turned out, one would simply see the cockroach circling a central point with a tangential speed v_{tan}. His absolute rotational velocity would be $\Omega_{XY} = \frac{v_{tan}}{r_p}$. As can be seen, $\vec{a}_n$ and $\vec{a}_{Cor}$ are aligned and point from the cockroach toward point O. These, then, added together, should produce the normal acceleration that one would get using Ω_{XY} instead to calculate it.

That is,

$$\vec{a}_n + \vec{a}_{Cor} = \Omega^2 r_p + 2\Omega v_{xy}$$

But just considering the motion of the bug alone

$$\vec{a}_{n-XY} = \Omega_{XY}^2 r_p = \left(\frac{v_{tan}}{r_p}\right)^2 r_p = \frac{\left(\Omega r_p + v_{xy}\right)^2}{r_p} = \Omega^2 r_p + 2\Omega v_{xy} + \frac{v_{xy}^2}{r_p}$$

Thus, it looks as if there is a discrepancy between the two calculations, since the latter one includes the extra term v_{xy}^2/r_p. Where does this term come from? The answer is found by considering what the bug's acceleration would be, walking on his circular path when Ω = 0. His normal acceleration on a non-spinning disk would be precisely v_{xy}^2/r_p. Thus $\vec{a}_n + \vec{a}_{Cor}$ does not include the local acceleration, $\vec{a}_{xy} = v_{xy}^2/r_p$ due simply to the cockroach's walking a circular path.

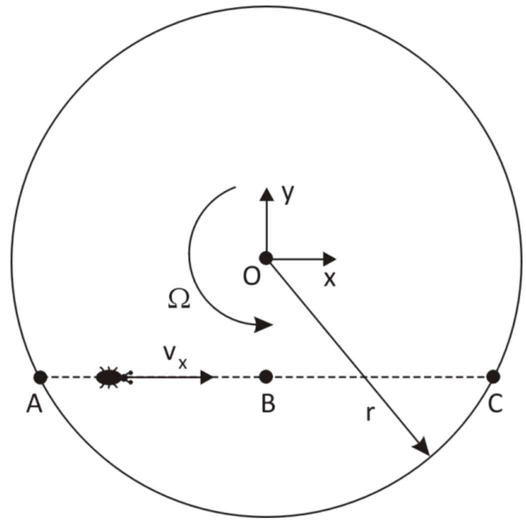
At right is shown the case where the cockroach walks a straight path across the disk from A to B to C, but it does not pass through the center of the disk. From the vector expression for $\vec{a}_{Cor}$,

$$\vec{a}_{Cor} = 2\vec{\Omega} \times \vec{v}_{xy}$$

nothing has changed. For this case, with a straight path parallel with the *x*-axis, $\vec{v}_{xy} = v_x\hat{\imath}$. Since $\vec{\Omega} = \Omega\hat{k}$, the cross produce yields a vector

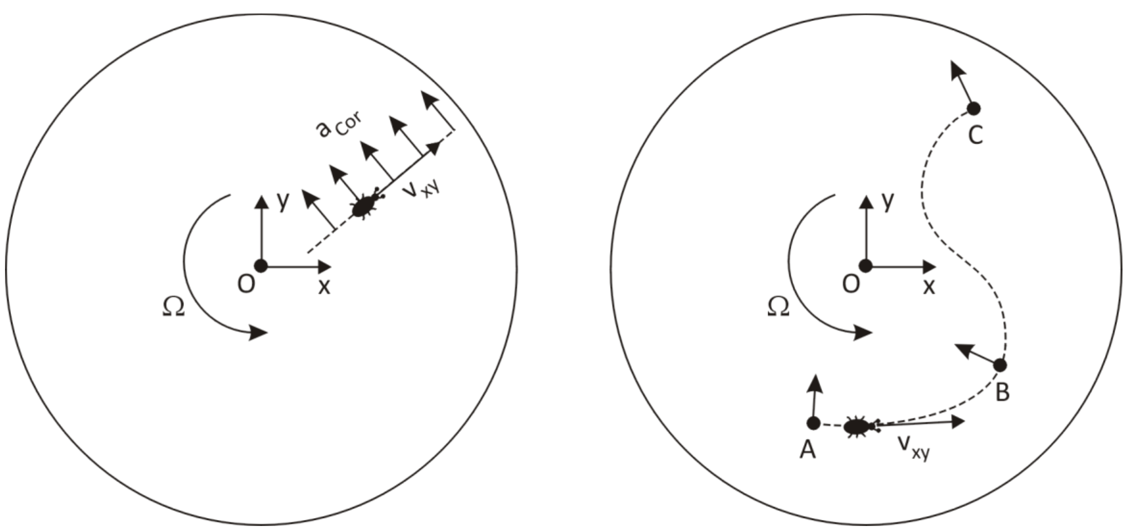
$$\vec{a}_{Cor} = 2\Omega\hat{k} \times v_x\hat{\imath} = 2\Omega v_x\hat{\jmath}$$

As stated in the last paragraph of the section above, with counter-clockwise rotation, $\vec{a}_{Cor}$ is always just 90° to the left of the path. Notice that neither $\vec{\Omega}$ nor $\vec{v}_{xy}$ change for any point on the path.



Cockroach walking on offset path

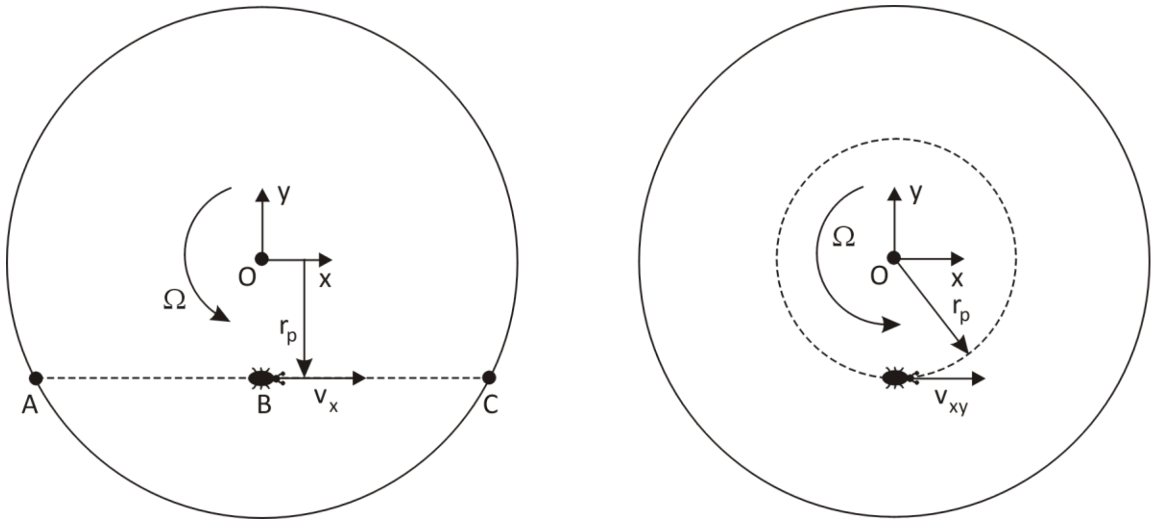
It is a mistake to think that Coriolis acceleration always is directed tangentially. This can be seen clearly in this case. At B in fact the Coriolis acceleration is directed normally, toward the center of rotation of the disk. The Coriolis acceleration is thus only tangentially directed when the cockroach's path passes through the center of rotation. For some random path, as shown below, the Coriolis acceleration is always directed to the left of the path. The path does not even have to be straight. The Coriolis acceleration is always normal to the path, to the left if the rotation is counter-clockwise, because of the cross product.



Random paths on spinning disk

It is interesting to compare the two cases, already presented, shown in the figure below. Let $v_x = v_{xy}$, that is, in both cases the cockroach is walking at the same pace. The acceleration at B in the left-hand drawing is

$$\vec{a}_n + \vec{a}_{Cor} = \Omega^2 r_p + 2\Omega v_{xy}$$



Comparison of accelerations on a straight path and a circular path

The acceleration in the right-hand drawing is instead

$$\vec{a}_n + \vec{a}_{Cor} + \vec{a}_{xy} = \Omega^2 r_p + 2\Omega v_{xy} + \frac{v_{xy}^2}{r_p}$$

What's interesting about this is that at B, the tangential velocity of the cockroach in both cases is the same. But the difference between the two can be seen in the case of a non-spinning disk. Even with no rotation, the cockroach on the right-hand disk experiences an acceleration toward point O, because of his circular path. The cockroach on the left-hand disk, on the other hand, would experience no acceleration, were Ω = 0.